Investigating Number Sense, Addition, and Subtraction

Unit Overviews for Grades K–3

Investigating Number Sense, Addition, and Subtraction (Grades K–3) is organized around 8 units. Each unit is developed around carefully-crafted contexts—realistic and fictional— and comprises a two-week (10-day) sequence of investigations, games, routines, and minilessons.

1 Bunk Beds and Apple Boxes: Early Number Sense

by Catherine Twomey Fosnot

This unit begins with the story of a pajama party—a sleepover during which eight children play, moving up and down bunk beds, teasing the babysitter who imagines she is losing and then gaining children!

The unit introduces the arithmetic rack as a powerful model and tool to act out the story. The arithmetic rack is a calculating frame consisting of two rows of ten beads with two sets of fives in each row. (Instructions for creating or buying your own arithmetic racks are included.) The five-structure of this apparatus supports the development of part-whole relations in early number sense. Since five is an amount that can often be perceived as a whole, it can be used to support understanding 6 as 5 + 1, or 4 as 5 – 1. It also supports the strategies of doubles and near doubles, 6 + 7 = 6 + 6 + 1, and making tens, 9 + 6 = 10 + 5. In this unit, children move the beads on the arithmetic rack to illustrate and develop an understanding that eight can be named in many ways, for example as 7 + 1, 5 + 3, or 4 + 4. The unit also includes the game Up and Down the Ladder, and employs the use of quick images (a series of related problems flashed only for seconds) to further develop early number sense.

As the unit progresses, the context shifts to an exploration of apple boxes. Children investigate the number of unique combinations for five apples of two kinds, green and red, and record the combinations for a grocer who is confused about how many arrangements there can be. In contrast to the bunk beds investigation in which children can easily imagine someone going up and down the ladder, now they must exchange. That is, instead of moving a counter to another group, the counter must be removed and replaced. This action is more difficult. The recording sheet for the grocer is designed in such a way that a staircase pattern emerges as one red apple is traded for a green apple each time. Boxes holding various numbers of apples (such as six, seven, eight, nine, and ten) are then explored to examine if the staircase pattern will always occur. Data are also collected on the number of possible arrangements for each box: a box of six apples has five possible arrangements; a box of seven has six arrangements; a box of ten has nine arrangements. This supports the development of a systematic way of producing all the possible arrangements and produces another inquiry: Can we predict the number of arrangements if we know the size of the box?

The unit ends with the Part-Whole Bingo game. This game can be played throughout the year as a way for children to extend composing and decomposing strategies as they establish equivalence, for example representing 7 as 5 + 2 or as 3 + 4, or even as 2 + 2 + 2 + 1.

2 Beads and Shoes, Making Twos: Extending Number Sense

by Madeline Chang and Catherine Twomey Fosnot

This unit begins with the context of walking in line—two lines of children holding hands. The context encourages children to explore doubles while also strengthening their understanding of one-to-one correspondence. As the unit progresses, children explore containers that could hold doubles (such as egg cartons, English muffin packages, and juice boxes). Then the context shifts to an examination of pairs of shoes for varying numbers of people. As children investigate these situations, they explore both pairing and doubling—for instance, how six pairs of shoes can also be seen as six right shoes plus six left shoes (six sets of two or two sets of six). Later children work with larger numbers and the terminology of odds and evens is introduced.

In the second week, the story Grandma's Necklaces is used to develop a context for several investigations related to patterns made with two colors. The first necklace (one blue/one green repeating) can only be made with an even number of objects, because the unit that repeats has two objects. The second necklace (five blue/five green repeating) and the third necklace (three blue/three green repeating) challenge children to see a group of objects doubled as the unit that repeats.

Minilessons in the unit are crafted to support the automatizing of doubles and their use in solving near doubles—for example, using 6 + 6 to solve 6 + 7, or 10 + 10 to solve 9 + 10. Quick images and the arithmetic rack are both used with strings of related problems. The unit also includes the Shoe Game. This game can be played throughout the year for further support in developing the use of doubles as an addition strategy.

3 The Double Decker Bus: Early Addition and Subtraction

by Maarten Dolk, Nina Liu, and Catherine Twomey Fosnot

This unit begins with the story of a double-decker bus—a bus that has two decks with ten seats on each. Five seats on each deck are red and five seats are white. The bus goes by quickly and the little girl in the story, sitting at her bedroom window and watching, works out ways to use the colors of the seats to calculate quickly how many people are on the bus. Her father drives a double-decker bus and she helps him figure out a way to know how many empty seats there are on the top deck even though he can't see them.

The unit introduces the arithmetic rack as a powerful model and tool to act out the story. The arithmetic rack is a calculating frame consisting of two rows of ten beads—two sets of five (one red and one white) in each row. (Instructions for creating or buying your own arithmetic racks are included.)

Cognitive psychologists, such as Susan Carey and Stanislas Dehaene (1999), have shown that even toddlers can recognize small amounts, such as two or three, as a unit and that this ability (known as "subitizing") is probably innate. Children can even do addition and subtraction with amounts of this size because they use this innate perceptual ability to see that three is one more than two. Using the arithmetic rack allows kindergarteners and first graders to build on their natural ability and see five as a unit. When five can be subitized as a whole, it can be used to support understanding of 6 as 5 + 1, 8 as 5 + 3, or 4 as 5 – 1. The arithmetic rack also supports the strategies of doubles and near doubles, 6 + 7 = 6 + 6 + 1, and making tens, 9 + 6 = 10 + 5.

In this unit, children move the beads on the arithmetic rack to represent passengers going from one deck on the bus to the other, and sitting in various combinations in the red and white seats. This context supports the development of the understanding that numbers can be named in many ways, for example 10 as 6 + 4, 7 + 3, or 5 + 5. The unit also includes minilessons with quick images, and strings of related addition and subtraction problems solved with the arithmetic rack to help automatize the basic facts.

Several games—Passenger Pairs, Rack Pairs, and Passenger Combos—are also included in this unit. They can be played throughout the year as a way for children to extend composing and decomposing strategies as they establish equivalence—for example, representing 7 as 5 + 2, 3 + 4, or 1 + 6 (Treffers, 1991).

4 Organizing and Collecting: The Number System

by Nina Liu, Maarten Dolk, and Catherine Twomey Fosnot

This unit begins with the story of the Masloppy family—an endearing, large family that finds it difficult to keep track of things. Everyone is forever losing, misplacing, and looking for things. One of the children, Nicholas, decides to sort, organize, and take inventory of things in the house. He counts and bundles materials and labels containers and baskets, and life in the Masloppy household is smoother thereafter.

The idea of taking inventory is brought to the classroom, where children work to count and label baskets of supplies and materials. The discussion focuses on organizing in groups and skip-counting, then specifically on groups of ten. The concept of place value is developed as the children pack objects into groups of ten and study patterns in the data. There are opportunities to deepen their understanding by packing in fives and playing games that focus on groups of ten, and there are minilessons that use the ten-frame as a visual model of five and ten. (The structure of the ten-frame is similar to that of the arithmetic rack, which is discussed in The *Double-Decker Bus* above. Both units use five and ten as landmark numbers: the arithmetic rack has four groups of five arranged in two rows of ten, the ten frame has two groups of five. The ten-frame is used in this unit because it resembles the context of packing more closely.) In the second week the inventory context is extended to include ordering more classroom supplies as a way to develop and support addition strategies, which include jumping to friendly numbers (multiples of ten) and jumping by ten.

As children pack and count groups of items, they begin to unitize—to count groups and objects at the same time. Children develop an understanding of place value as they construct the idea that the number of packs and loose items is related to the total number of objects and that the numbers change when items are added to make full packs or when a pack of ten is added.

The game Collecting Stamps and its variations are included in this unit. These can be played throughout the year as a way for children to develop place value and addition strategies. The game and its variations extend composing and decomposing strategies while promoting understanding of equivalence—for example, representing 26 + 8 as equal to 26 + 4 + 4.

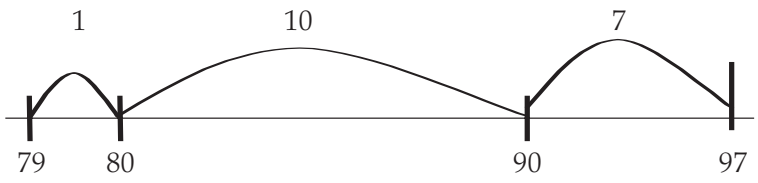
5 Measuring for the Art Show: Addition on the Open Number Line and Subtraction

BY CATHERINE TWOMEY FOSNOT

The focus of this unit is the development of the open number line model within the context of measurement. As the unit progresses, the number line is used as a model for double-digit addition strategies. The unit begins with the story of a teacher who has offered to organize an art show of children's work as a school fund-raiser. The children have produced beautiful pieces of art and the teacher and several children set out to make signs to hang underneath each piece, listing the title of the piece, the artist's name, and the price. They want to measure each art piece very carefully so that the sign will be exactly the same length as the piece of art. But this huge pile of work is daunting. Thankfully, the students soon figure out a solution. They sort the art by size, measure each size, and make a blueprint—a pattern strip—that will be used for cutting all the signs.

The story sets the context for a series of investigations in this unit. Children measure various sizes of art paper with connecting cubes and then place the measurements onto a long strip of adding machine paper, to be used as a blueprint or pattern for cutting the signs. As the unit progresses, lengths of fives and tens are introduced in place of the cubes and the blueprint is progressively developed into an open number line—a helpful model used as a tool to explore and represent strategies for double-digit addition.

In contrast to a number line with counting numbers written below, an "open" number line is just an empty line used to record children's addition (and later subtraction) strategies. Only the numbers children use are recorded and the addition is recorded as leaps or jumps. For example, if a child's strategy for adding 18 + 79 is to keep 79 whole and decompose the 18 into smaller pieces, moving to a landmark number of 80 (79 + 1 + 10 + 7), it would be recorded on the open number line like this:



Such representations help children move beyond tedious strategies like counting one by one to strategies such as taking leaps of ten, splitting, and using landmark numbers.

Several minilessons for addition are also included in the unit. These are structured as strings of related problems designed to guide learners more explicitly toward computational fluency with double-digit addition.

The unit culminates with an art show. Thus, as you progress through the unit, you may find it helpful to work with the art teacher in your school to collect pieces of student artwork.

6 Trades, Jumps, and Stops: Early Algebra

by Catherine Twomey Fosnot and Patricia Lent

The story *The Masloppy Family Goes to New York City* sets the stage in this unit for a series of investigations to develop several big ideas and strategies important in the algebra strand. Seven-year-old Nicholas Masloppy (fondly known as the Organizer) and his brother and sisters are all waiting for the very special night when the family's big piggy bank will be opened. The family has been saving for a long time and now the bank is full. They are hoping to have enough money to go to New York City, where they will ride the subway to the Empire State Building, take a boat ride around the city, and visit the American Museum of Natural History. When the bank is opened, Nicholas's task is to organize the money into three equivalent piles for the three excursions.

The piggy bank context is developed in the story and then used in the unit as an important model for exchange and equivalence. The coins in the bank cannot be distributed into three piles evenly because not all of the coins are in multiples of three. Children need to redistribute and exchange coins in order to make three equivalent amounts. As the unit progresses, the piggy bank context is used to introduce and analyze equations and to develop strategies for simplifying them, such as using the associative and commutative properties, "canceling," and substituting. Variables are introduced with the additional context of foreign coins of unknown denominations.

As the unit progresses, the context of subway stops at which numbers of passengers board and detrain is used to explore net change and functions. Equivalent expressions are generated as ways to describe the changes and children work to develop convincing proofs that they have found all the possible ways.

Several minilessons for algebra are also included in the unit. These are structured initially as a game of "twenty questions" to determine the denominations of hidden coins totaling 50 cents and later as strings of related problems. Initially the focus of the minilessons is on equivalent trades and writing mathematical statements using the relational signs $<$, $>$, and $=$. As the unit progresses, the minilessons support the development of an understanding of the commutative and associative properties of addition, and of strategies for simplifying equations and solving for unknowns (focusing on strategies such as "canceling," substituting using equivalence, and undoing.)

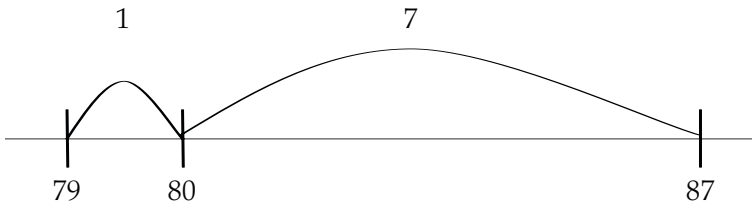
7 Ages and Timelines: Subtraction on the Open Number Line

by Catherine Twomey Fosnot

This unit begins with the story of Carlos, an eight-year-old boy who is fascinated by his great-grandfather's thick, beautiful silver hair. His great-grandfather lives in Puerto Rico and Carlos is preparing to meet him for the first time. Having only seen photos of him as a much younger man, Carlos wonders how old his great-grandfather is and how many years it will take before he might have hair like that, too. As Carlos begins to investigate these questions, his whole family becomes involved in exploring age differences and figuring out how old they each were when Carlos was born. When Carlos shares his investigation with his teacher, the whole school gets involved in the project.

This story context sets the stage for a series of investigations in this unit. Children interview their family members and compare age differences. Timelines are introduced as a context for using the open number line—a helpful model used as a tool to explore and represent strategies for addition and subtraction. This unit will focus on the open number line as a model for subtraction.

In contrast to a number line with counting numbers written below, an "open" number line is just an empty line used to record children's addition and subtraction strategies. Only the numbers that children use are recorded and the addition and subtraction are recorded as leaps or jumps. For example, if a child's strategy for adding 8 + 79 is 79 + 1 + 7, using a landmark number of 80, it would be recorded on the open number line as:



The recording would be similar if a child solves 87 – 8 by first removing 7 and then 1. Modeling children's thinking on the open number line helps them move beyond counting one by one for addition and subtraction, to strategies such as taking leaps of ten, decomposing, and/or using landmark numbers. Use of the open number line also encourages discussion of the relationship between addition and subtraction and of the relationship between various problems in which the operation of subtraction can be employed—such as removal, comparative difference, and finding a missing addend.

As the unit progresses, timelines are used to record years of birth, rather than ages. This change in context challenges learners to grapple with larger numbers and with the changing places of the part-whole relations of numbers on the number line. For example, first the number 79 may be marked on the number line as 8 less than 87; then it may be the difference between 2005 and 1926. Several minilessons for subtraction are also included in the unit. These are structured using strings of related problems as a way to guide learners more explicitly toward computational fluency with subtraction.

Note: It is expected that children will have had substantial experience with number lines prior to this unit. If this is not the case, you might want to take a look at the unit *Measuring for the Art Show,* to see how the number line model can be developed.

8 The T-Shirt Factory: Place Value, Addition, and Subtraction

BY CATHERINE TWOMEY FOSNOT

This unit begins with the story of *Grandma Eudora's T-Shirt Factory.* Grandma Eudora is part of the Masloppy family—a large, endearing family that finds it difficult to keep track of things. Everyone is forever losing, misplacing, and looking for things. One of the children, Nicholas, decides to sort, organize, and take inventory of things in the house, including Uncle Lloyd's T-shirts, which he arranges in rolls with rubber bands. One day as Uncle Lloyd is doing the laundry, Itchy, the family dog, knocks over a bottle of bleach. The result of this mishap is colorful tie-dyed T-shirts, which Grandma begins to sell in a highly successful business—Grandma Eudora's T-Shirt Factory.

The idea of the T-shirt factory is brought to the classroom as a simulation. Children work in groups (companies with factories) making and selling T-shirts and organizing their warehouses. The main focus of the unit is place value, regrouping, equivalence, and the recording of the inventory. Students keep track of inventory before and after shipping orders, as boxes and rolls in the warehouse are opened so that orders can be filled. Within the context of the need for pencil-and-paper recordings of transactions, the standard addition and subtraction algorithms are explored. The concept of place value is developed to three and four places during the simulation as children organize the warehouse: packing rolls of T-shirts (ten to a roll) in storage boxes that hold ten rolls and calculating the income from sales of T-shirts at $10 each. Students, playing the role of employees, keep accounting ledgers to record the sales of specific sizes and the total company inventory.